GERMAN - MONGOLIAN INSTITUTE FOR RESOURCES AND TECHNOLOGY

The present work was submitted to
the German-Mongolian Institute of Resources and Technology

Design and Optimisation of an On-Site Greywater Recycling System for a Manufacturing Facility in Ulaanbaatar, Mongolia

Bachelor's Thesis

By

Tuguldur Enkhbayar

Study program: Industrial engineering

Student ID: B2200569

1st Supervisor/Examiner: Prof. Dr. Simon Glöser-Chahoud

2nd Supervisor/Examiner: Dr. Bolormaa Purevjav

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List of Abbreviations and Symbols

BOD	Biochemical Oxygen Demand
CAPEX	Capital Expenditure
CHPP4	Combined Heat and Power Plant No. 4
CIP	Clean-in-Place
D(t)	Daily non-potable water demand (m³/day)
DEL(t)	Delivered water volume on day t (m³)
EQ(t)	Equalization tank level on day t (m³)
G(t)	Daily greywater generation (m³/day)
GMIT	German-Mongolian Institute for Resources and Technology
ISO	International Organisation for Standardisation
LCCA	Life-Cycle Cost Analysis
MC	Monte Carlo
MNT	Mongolian Tugrik (currency)
NPV	Net Present Value (MNT)
NTU	Nephelometric Turbidity Unit
OPEX	Operating Expenditure
OV_eq(t)	EQ tank overflow volume on day t (m³)
OV_tr(t)	Treated storage overflow volume on day t (m³)
P5	5th percentile reliability across Monte Carlo runs
PFD	Process Flow Diagram
Q_treat	Maximum daily treatment capacity (m³/day)
R	System reliability (fraction of days demand is met)
S_max	Treated water storage tank capacity (m³)
TR(t)	Treated water storage tank level on day t (m³)
TSS	Total Suspended Solids
UV	Ultraviolet
V_eq	Equalization/sedimentation tank capacity (m³)
W(t)	Treatment throughput on day t (m³/day)

WHO	World Health Organization
η	Treatment efficiency factor (dimensionless)
μ_G	Mean daily greywater generation (m³/day)
μ_D	Mean daily non-potable demand (m³/day)
σ_G	Standard deviation of daily generation (m³/day)
σ_D	Standard deviation of daily demand (m³/day)

ABSTRACT

This study presents the technical design, stochastic simulation, and economic optimisation of an on-site greywater recycling system for a representative manufacturing facility in Ulaanbaatar, Mongolia. Driven by accelerating water scarcity and the strategic objectives of the Millennium Challenge Corporation (MCC) Mongolia Water Compact, the research develops a five-unit treatment train (screening → equalization/sedimentation → sand filtration → UV disinfection → treated storage), constructs a physically rigorous two-tank daily water-balance simulation with Monte Carlo analysis (100 stochastic scenarios per configuration), conducts a Life-Cycle Cost Analysis (LCCA) over 20 years including equipment replacement costs and salvage value, and employs a three-dimensional combinatorial optimisation across 4,560 configurations to identify the most cost-effective and reliable design. The simulation explicitly models the Equalization/Sedimentation Tank (TK-101) and the Treated Water Storage Tank (TK-102) as independent daily state variables with treatment efficiency ($\eta = 0.95$), maintenance downtime, and realistic weekend and holiday demand patterns, accurately capturing the hydraulic buffering dynamics that single-tank simplifications fail to represent. The optimal configuration — $V_{eq} = 6$ m³ equalization tank, $Q_{treat} = 40$ m³/day treatment capacity, $S_{max} = 12$ m³ treated storage — achieves a mean reliability of 97.2% (P5 = 95.3% across Monte Carlo runs), a CAPEX of 94.8 million MNT, and a 20-year NPV of approximately −71.3 million MNT, with a payback period of approximately 3 years. These findings confirm that industrial greywater reuse is technically robust and economically compelling under Mongolian conditions, and the framework is designed to accept real facility data with minimal structural modification.

Keywords: greywater recycling; two-tank water balance; Monte Carlo simulation; life-cycle cost analysis; industrial water reuse; Ulaanbaatar; Mongolia; equalization tank; stochastic modelling; combinatorial optimisation.

1. Introduction

1.1 Background and Problem Statement

Ulaanbaatar, the capital of Mongolia, faces chronic and worsening water scarcity. The city's entire freshwater supply depends on finite groundwater aquifers beneath the Tuul River basin, which are being depleted at rates that substantially exceed natural recharge under current extraction regimes. This crisis is compounded by three concurrent pressures: rapid industrial expansion concentrated in the city's western and southern districts; population growth driven by ger district urbanisation; and the intensifying effects of the dzud cycle — Mongolia's periodic extreme winter–summer climate oscillation — which reduces summer recharge while increasing winter water demand for heating-related purposes.

Manufacturing facilities are among the largest single-category water consumers in Ulaanbaatar's industrial sector. They simultaneously generate substantial volumes of low-contamination process effluent from non-production streams. This effluent — termed greywater — includes washdown water from equipment and floor cleaning operations, cooling tower blowdown, and sanitary water from non-toilet fixtures. Its key characteristic is that it requires only moderate treatment to reach a quality level suitable for non-potable reuse: floor washing, cooling tower makeup water, dust suppression on unpaved surfaces, and fire suppression reserves. None of these end uses require drinking-water-grade quality, and all are currently consuming expensive, treated groundwater supply that could be substituted by recycled greywater at a fraction of the full treatment cost.

The Millennium Challenge Corporation (MCC) Mongolia Water Compact, a bilateral agreement between the United States and Mongolian governments, has explicitly identified industrial wastewater reuse as a strategic priority for Ulaanbaatar's water security. Despite this policy mandate and the obvious economic case, the majority of industrial facilities in the region have not implemented on-site recycling systems. The barriers are partly technical — uncertainty about appropriate treatment train design, sizing methodology, and local performance benchmarks — and partly economic — limited access to the investment capital required and unclear return-on-investment projections under Mongolia's historically subsidised water tariff structure.

This thesis directly addresses these barriers by designing, simulating, and optimising an on-site greywater recycling system from first principles, calibrated to representative Mongolian industrial parameters.

Mongolia faces a unique combination of water stress factors that distinguish it from other Central Asian countries. The nation's per capita freshwater availability, while statistically adequate at the national level, is severely concentrated in the northern river basins, leaving the Gobi region and Ulaanbaatar's rapidly growing industrial zone dependent on groundwater extraction that increasingly exceeds natural recharge rates. The Tuul River, which historically supplied much of Ulaanbaatar's water, has experienced significant flow reductions over the past two decades due to the combined effects of upstream mining operations, reduced precipitation, and permafrost degradation [11]. These hydrological pressures are compounded by Mongolia's extreme continental climate, which subjects water infrastructure to thermal cycling between −40°C in winter and +35°C in summer, creating engineering challenges for water treatment and storage systems that are not encountered in the temperate climates where most greywater reuse research has been conducted.

The economic context is equally distinctive. Mongolia's industrial water tariff structure combines a volumetric supply charge with a separate discharge fee, creating a double economic incentive for water recycling: each cubic metre recycled avoids both the purchase cost and the discharge cost. The Green Group industrial complex in Bayangol District, which serves as the case study for this research, paid an effective tariff of 5,294 MNT per cubic metre in 2025, resulting in an annual water expenditure of 136.9 million MNT across its five co-located subsidiaries. At this tariff level, even modest recycling rates can generate substantial cost savings, making greywater reuse economically attractive even without environmental subsidies or regulatory mandates.

The regulatory environment is also evolving. Mongolia's Green Development Policy, adopted in 2014, established targets for industrial water efficiency improvements and waste reduction. More recently, the World Bank's facilitation of Mongolia's first industrial water recycling scheme between the Amgalan Thermal Plant and MCS Coca-Cola in Ulaanbaatar [10] has demonstrated institutional willingness to support water reuse initiatives. However, on-site treatment and reuse at the individual facility level — as proposed in this thesis — remains largely unexplored in the Mongolian context, representing both a technical gap and a policy opportunity.

In Mongolia specifically, the concept of industrial water recycling is at an early developmental stage. The MCS Coca-Cola project, facilitated by the World Bank in 2026, represented a landmark inter-facility water transfer scheme but did not involve on-site treatment technology or simulation-based design optimisation [10]. Previous assessments of Mongolian industrial water use have focused primarily on the mining sector, where water consumption is orders of magnitude larger and the regulatory framework is more developed. The manufacturing sector, despite its growing economic significance and concentration in the Ulaanbaatar industrial zones, has received comparatively little attention in the water management literature.

The application of stochastic simulation methods to water system design has gained traction in the broader water engineering literature. Monte Carlo simulation, in particular, has been recognised as a robust approach for quantifying performance uncertainty in systems subject to variable inputs [12]. However, its application to greywater recycling system design remains limited. Most existing studies either use deterministic approaches with fixed daily flows or apply stochastic methods to rainwater harvesting rather than greywater reuse. This study bridges this gap by integrating Monte Carlo simulation with systematic design optimisation in a greywater-specific context.

However, existing studies on greywater reuse systems predominantly rely on simplified modelling approaches that do not explicitly represent the hydraulic buffering dynamics between equalization and storage tanks, creating a gap in accurately assessing system reliability and optimising design configurations. For instance, Knutsson and Knutsson (2021) conceptualised the treatment process as a single-tank mass balance system and acknowledged that such approaches lack the fidelity needed for technology optimisation. Similarly, Maskwa et al. (2021) combined system dynamics modelling with life-cycle cost assessment across 12 U.S. cities but relied on deterministic historical data without stochastic uncertainty quantification. These limitations motivate the development of a more physically rigorous simulation framework.

1.2 Research Aim

The aim of this study is to design, simulate, and economically evaluate an on-site greywater recycling system for manufacturing facilities in Mongolia, using a physically rigorous two-tank modelling approach that explicitly represents

equalization and treated-water storage as separate hydraulic state variables, thereby improving the accuracy of reliability assessment and enabling systematic design optimisation under stochastic operating conditions.

1.3 Research Objectives

The following objectives are formulated to operationalise the research aim through a structured engineering and analytical approach. Five specific, measurable objectives guide this research:

- Design a technically appropriate multi-unit treatment train — comprising screening, equalization/sedimentation, sand filtration, and UV disinfection — and document it with a formal Process Flow Diagram (PFD) using standard engineering notation.
- Construct a stochastic daily water-balance simulation incorporating two physically distinct tank state variables (TK-101 and TK-102) and seasonal demand and generation variability appropriate for the Mongolian continental climate.
- Assess the 20-year economic viability of the system through a Life-Cycle Cost Analysis (LCCA) encompassing capital expenditure (CAPEX), operating expenditure (OPEX), and water-cost avoidance savings.
- Optimise the system configuration via a three-dimensional combinatorial search over equalization tank volume (V_eq), treatment capacity (Q_treat), and treated storage volume (S_max), subject to a minimum system reliability constraint of R ≥ 95%.
- Formulate actionable policy recommendations aligned with Mongolian water regulations and the MCC Water Compact objectives.

1.4 Research Questions

This study addresses the following research questions:

RQ1. How can an on-site greywater recycling system be appropriately designed for manufacturing facilities under Mongolian conditions?

RQ2. How does a stochastic two-tank water-balance model represent system performance under variable generation and demand conditions?

RQ3. What is the economic feasibility of the greywater recycling system over its lifecycle?

RQ4. What system configuration minimises lifecycle cost while maintaining required reliability?

RQ5. What are the practical and policy implications of implementing greywater recycling systems in Mongolia?

1.5 Significance of the Study

The principal academic contribution of this work is the physically rigorous two-tank simulation architecture. Published industrial greywater feasibility studies — including influential work by Friedler & Gross (2008) and more recent optimisation studies by Rodríguez-Gómez et al. (2025) — predominantly employ single-tank mass balance models in which equalization and storage are conflated. This simplification systematically understates system reliability and misrepresents the value of treatment capacity, because it fails to model the buffering of peak generation days by the equalization tank. The present work demonstrates that the equalization tank (TK-101) and treated water storage tank (TK-102) must be simulated as separate state variables to correctly capture the system's hydraulic behaviour, and that this distinction materially affects the optimisation outcome.

This study contributes both a methodological advancement in system modelling — through the explicit two-tank stochastic simulation architecture — and a practical decision-support framework for industrial water reuse under data-constrained conditions typical of developing-economy manufacturing sectors.

2. Literature Review

2.1 Greywater Reuse in Industrial Systems

Greywater reuse has been widely studied as a water conservation strategy in both domestic and industrial contexts, with treatment systems typically combining physical, biological, and disinfection processes depending on reuse requirements. In industrial settings, greywater generated from facility cleaning, cooling systems, and employee amenities represents a significant volume of potentially reusable water that is conventionally discharged to municipal sewer systems at considerable cost. The separation and on-site treatment of this water has been recognised as a key strategy for reducing freshwater consumption and wastewater discharge volumes, particularly in water-stressed regions where groundwater extraction rates exceed natural recharge capacity.

Industrial greywater reuse differs from residential applications in several important respects. Generation volumes are typically higher and more concentrated during working hours, demand patterns are driven by production schedules rather than occupancy, and seasonal variations may be more pronounced due to climate-dependent cooling or heating requirements. These characteristics necessitate careful system design that accounts for temporal variability in both supply and demand — a consideration that is inadequately addressed by static design methods based on average daily flows.

2.2 Treatment Technologies for Non-Potable Reuse

Conventional treatment trains for non-potable greywater reuse typically combine physical pre-treatment (screening and sedimentation), filtration (sand filters or membrane systems), and disinfection (ultraviolet irradiation, chlorination, or ozone). Sand filtration combined with UV disinfection is widely applied in non-potable reuse systems due to operational simplicity and cost-effectiveness, achieving BOD removal rates of 75–89%, TSS removal of 61–97%, turbidity reduction to below 5 NTU, and 3 log units of pathogen inactivation (Lazarova and Asano, 2014).

Membrane-based systems — including microfiltration (MF) and membrane bioreactors (MBR) — offer higher effluent quality but at substantially increased capital and operational costs. Rodríguez et al. (2025) compared microfiltration and rotating biological contactor (RBC) technologies for greywater reuse in Chile, finding that treatment technology selection significantly influences system

economics. For non-potable industrial applications such as irrigation, facility cleaning, and cooling water makeup, conventional sand filtration and UV disinfection are generally sufficient to meet applicable water quality standards, making them the appropriate technology choice for cost-sensitive industrial applications in developing economies.

The performance characteristics of sand filtration for greywater treatment have been extensively documented. Pidou et al. [19] reviewed treatment options for greywater recycling and reported that slow sand filters achieve BOD removal of 85–95% and turbidity reduction to below 2 NTU under optimal loading conditions, while rapid sand filters operate at higher throughput rates with slightly lower removal efficiencies (75–89% BOD removal). The choice between slow and rapid sand filtration depends on the available footprint and the required treatment capacity: for industrial applications with generation rates exceeding 20 m^3/day, rapid sand filtration is typically preferred due to the smaller filter bed area required.

Ultraviolet (UV) disinfection provides effective pathogen inactivation without the formation of disinfection by-products associated with chlorination. The UV dose required for 3 log (99.9%) inactivation of E. coli is approximately 10 mJ/cm^2, well within the capacity of medium-pressure UV lamps operating at the design flow rates considered in this study [13]. The principal limitation of UV disinfection is its sensitivity to water turbidity: suspended particles can shield pathogens from UV exposure, necessitating effective upstream filtration. This sequential dependency between the sand filter and the UV unit is captured in the simulation model through the treatment efficiency parameter $\eta = 0.95$, which represents the combined probability of adequate turbidity reduction and effective UV exposure.

Alternative disinfection methods including chlorination and ozonation were considered but rejected for this application. Chlorination, while effective and inexpensive, introduces residual chlorine that may be unsuitable for irrigation applications and requires careful dosing and monitoring. Ozonation provides excellent disinfection but at substantially higher capital and energy costs, with operating complexity that is disproportionate for medium-scale industrial applications. The sand filter plus UV combination was therefore selected as the most appropriate technology for the Green Group application, balancing treatment effectiveness, operational simplicity, and lifecycle cost.

Oteng-Peprah, Acheampong, and deVries [16] conducted a comprehensive review of greywater characteristics, treatment systems, and reuse strategies across 45 peer-reviewed studies. Their meta-analysis confirmed that greywater treatment systems combining physical filtration with disinfection consistently meet non-potable reuse standards for applications including toilet flushing (BOD < 10 mg/L, turbidity < 2 NTU), irrigation (fecal coliform < 200 CFU/100mL), and industrial cleaning. The treatment train proposed in this study is consistent with the best-performing configurations identified in their review, providing confidence that the proposed system will meet applicable water quality standards for the intended non-potable end uses at the Green Group facility.

Li, Wichmann, and Otterpohl [15] reviewed 42 technological approaches for greywater treatment and reuse, classifying systems by complexity, cost, and effluent quality. Their classification framework places the sand filter and UV disinfection combination in the “moderate complexity, moderate cost, high reliability” category, which aligns with the engineering requirements of the Green Group case study: the system must operate reliably with minimal operator intervention while maintaining treatment costs below the threshold at which recycling becomes economically unattractive.

2.3 Modelling Approaches for Greywater Systems

Previous studies have applied both deterministic and stochastic modelling approaches to evaluate greywater reuse systems. Knutsson and Knutsson (2021) developed a simulation model for a 29-apartment building in Gothenburg, Sweden, using real disaggregated water consumption data from ZigBee-enabled rotary vane meters. However, the treatment process was conceptualised as a single-tank mass balance system, which the authors acknowledged does not provide the fidelity needed for design optimisation. The model did not separately represent equalization and treated water storage, limiting its ability to capture hydraulic buffering effects under variable inflow conditions.

Maskwa, Gardner, and Mo (2021) combined system dynamics modelling with life-cycle cost assessment to evaluate household greywater recycling and rainwater harvesting systems across 12 cities in the United States. Their study identified optimal tank sizes of 2–3 m³ for multifamily housing and payback periods varying by city, but relied on 10 years of deterministic precipitation data without stochastic simulation to quantify reliability uncertainty. The model

employed a single-tank representation without explicit treatment rate constraints between system components.

Manteaux, Besson, and Sperandio (2024) modelled decentralised greywater treatment at the district scale for a mixed residential-office area in France, but assumed that greywater produced during a given day could only meet that same day's demand, without tank carry-over between days. This simplification ignores the buffering function of storage tanks, which is critical for matching variable supply with variable demand over multi-day periods.

Chuang et al. (2016) designed a customised greywater system for residential buildings in Taiwan and simulated a 20-year life-cycle cost. However, the study used fixed daily consumption figures derived from national averages (72 litres per person per day for toilet flushing) rather than stochastic daily variation, and did not incorporate seasonal patterns, weekend effects, or maintenance downtime into the analysis.

2.4 Economic Evaluation Methods

Life-Cycle Cost Analysis (LCCA) is the established methodology for assessing the economic feasibility of water reuse systems, incorporating capital expenditure (CAPEX), operational expenditure (OPEX), and avoided cost components. Net Present Value (NPV) is the most widely used metric for comparing system configurations over a defined project lifetime, as it accounts for the time value of money through discounting (Friedler and Gross, 2008). However, cost functions in early-stage feasibility studies are often simplified — typically linear in the sizing variable — and may not capture non-linear scaling effects from equipment standardisation and bulk purchasing.

The World Bank recently facilitated Mongolia's first industrial-scale water recycling scheme between the Amgalan Thermal Plant and MCS Coca-Cola in Ulaanbaatar, with a policy paper outlining regulatory pathways for scaling industrial water recycling nationwide (World Bank, 2026). However, this initiative focused on inter-facility water transfer rather than on-site treatment and reuse, and did not involve simulation-based design optimisation or stochastic performance assessment.

2.5 Research Gap

Despite extensive research on greywater reuse systems, several specific limitations remain in the existing literature that this study aims to address.

First, existing modelling approaches frequently simplify system hydraulics by combining equalization and storage into a single state variable, as demonstrated by Knutsson and Knutsson (2021) and Maskwa et al. (2021). This simplification can underestimate system reliability under variable inflow conditions because it fails to capture the treatment rate bottleneck between the equalization tank and the treated water storage tank. The sequencing of overflow checks relative to treatment withdrawal — which materially affects simulated water loss — has not been explicitly analysed in the greywater modelling literature.

Second, few studies integrate stochastic simulation with optimisation techniques to identify cost-effective system configurations. Maskwa et al. (2021) optimised tank sizes but used deterministic data, while Rodríguez et al. (2025) optimised treatment technology selection but did not model stochastic daily variation in generation and demand. No study identified in this review simultaneously optimises equalization tank volume, treatment plant capacity, and storage tank volume under Monte Carlo simulation with reliability-constrained NPV minimisation.

Third, there is limited application of such models in the Mongolian industrial context, where extreme climatic variability (seasonal temperature swings from −30°C to +35°C), specific water tariff structures, Mongolian work schedules including Tsagaan Sar and Naadam holidays, and an evolving regulatory framework significantly influence both system performance and economic viability.

This study addresses these gaps by developing a two-tank stochastic simulation framework integrated with economic optimisation, tailored to representative industrial conditions in Ulaanbaatar. The following chapter presents the methodology developed to address the identified research gaps.

3. Methodology

3.1 Research Design Overview

The methodology comprises four sequential, interdependent stages. Stage 1 (Treatment Train Design) establishes the physical system layout and component specifications through deterministic hydraulic sizing calculations. Stage 2 (Stochastic Water-Balance Simulation) evaluates dynamic daily performance over a 365-day horizon using Python-based numerical simulation. Stage 3 (Life-Cycle Cost Analysis) translates performance outcomes into a 20-year economic assessment. Stage 4 (Combinatorial Optimisation) searches the three-dimensional decision variable space to identify the configuration that minimises total lifecycle cost subject to the reliability constraint.

This research adopts a quantitative, simulation-based engineering approach, combining system design, stochastic modelling, and economic optimisation to evaluate system performance under uncertainty. The use of stochastic simulation is appropriate given the inherent variability in greywater generation and demand, which cannot be adequately captured through deterministic approaches that assume fixed daily flows.

The complete simulation and optimisation code is publicly available for inspection, reproduction, and adaptation:

Google Colab Notebook:
https://colab.research.google.com/drive/1WXoDiMfzKk8Ut2Vjl5oCKHy3JOa161jz?usp=sharing

3.2 Research Framework and Alignment

The research framework integrates system inputs (greywater generation and demand), treatment system design, stochastic simulation, and economic evaluation into a unified decision-support model. Inputs are processed through a two-tank water balance model, producing outputs in terms of system reliability, water reuse volume, and economic performance. These outputs are then evaluated through combinatorial optimisation to identify the most cost-effective system configuration meeting the reliability constraint.

The methodology is directly aligned with the research objectives and questions: system design addresses Objective 1 and RQ1; the stochastic two-tank

simulation addresses Objective 2 and RQ2; the Life-Cycle Cost Analysis addresses Objective 3 and RQ3; the three-dimensional optimisation addresses Objective 4 and RQ4; and the synthesis of technical and economic results into policy recommendations addresses Objective 5 and RQ5.

3.3 Case Study Facility and Input Parameters

The analysis is based on a representative light-to-medium manufacturing facility in the Ulaanbaatar metropolitan area. The facility operates a single shift (8–10 hours/day, 6 days/week) and generates greywater from three main non-process streams: (i) equipment and floor cleaning operations, which contribute the largest and most variable daily volume; (ii) cooling tower blowdown, which is highly seasonal and nearly absent in winter months; and (iii) sanitary non-blackwater sources (handwashing, janitorial), which are relatively constant year-round.

Mean daily greywater generation is estimated at $\mu_G = 41$ m³/day with a standard deviation of $\sigma_G = 5$ m³/day, and mean daily non-potable reuse demand at $\mu_D = 30$ m³/day with $\sigma_D = 4$ m³/day. These values are consistent with published benchmarks for comparable industrial facilities in Central Asian urban environments.

Data availability note: These parameters are benchmark estimates. Correspondence with candidate partner companies in Ulaanbaatar's industrial district is ongoing. Once facility-specific metered data are received, the model will be re-run with updated inputs and the results compared against the benchmark-based projections reported here.

As facility-specific data are not yet available, the study is positioned as a preliminary feasibility and methodological demonstration, with results intended to be refined upon acquisition of real operational data. The simulation model is architecturally independent of the specific input values — updating the input parameters requires no structural modification to the simulation or optimisation code.

Table 1: Calibrated Simulation Parameters from Green Group 2025 Data

Parameter	Calibrated Value	Source
Total site water consumption	70.8 m³/day	2025 billing records
Annual water cost (Green Group)	136.9 M MNT	2025 financial records
Effective water tariff	5,294 MNT/m³	Calculated from 2025 data
Site total area	80,000 m²	Site survey
Mean greywater generation μ_G	41 m³/day	~58% of treatable streams

Mean non-potable demand μ_D	30 m³/day	Floor + irrigation + equipment
Generation seasonal amplitude	±19.3%	Measured from 2025 monthly data
Demand seasonal amplitude	±30%	Driven by summer irrigation
Treatment efficiency η	0.95	Sand filter + UV literature

3.4 Process Flow Diagram and Treatment Train

3.4.1 Process Flow Diagram

Figure 1 presents the Process Flow Diagram (PFD) for the greywater recycling system, drawn using standard ISA instrumentation and equipment notation. The PFD defines the physical system that both the simulation model and the cost functions represent. All components are referenced by their equipment tags throughout this thesis.

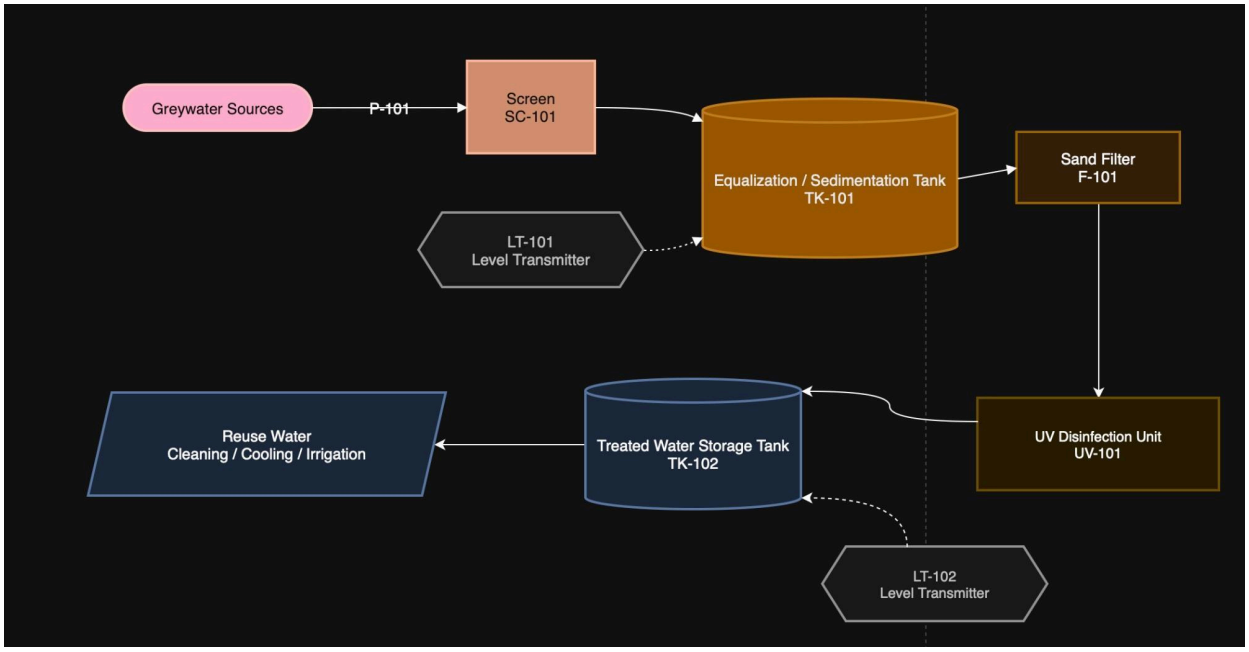


Figure 1. Process Flow Diagram (PFD) for the on-site greywater recycling system. Equipment notation follows ISA standards. Solid arrows indicate primary process flow; dashed arrows indicate instrumentation signals from level transmitters LT-101 and LT-102.

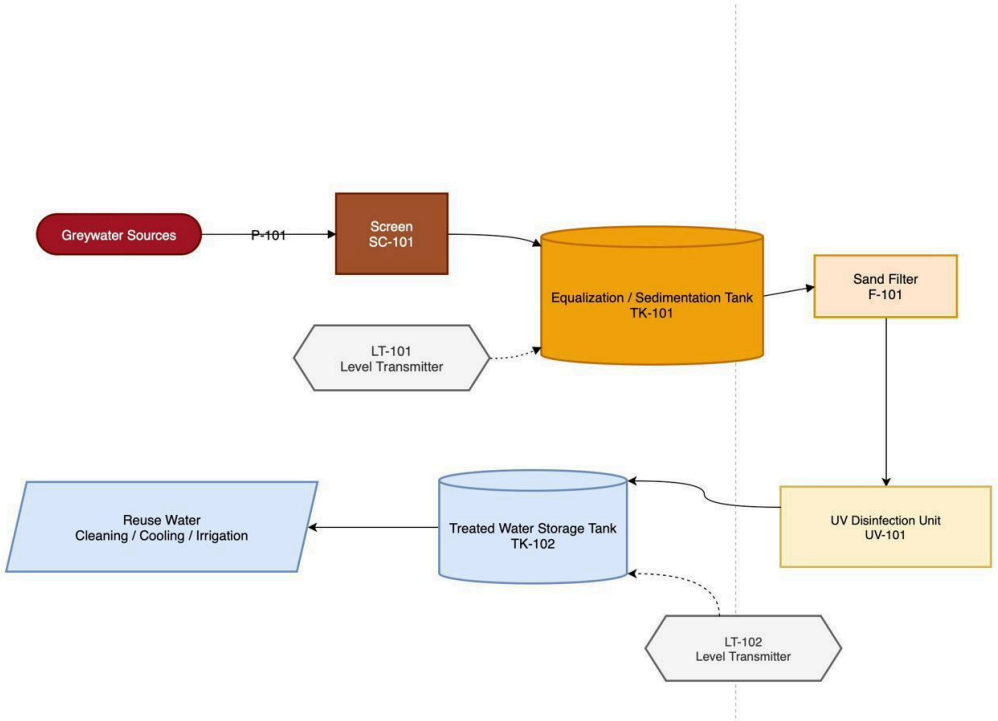


Figure 8: Process Flow Diagram (PFD) of the proposed two-tank greywater recycling system. Greywater from facility sources is collected and pumped (P-101) through a coarse screen (SC-101, 2 mm mesh) into the Equalization and Sedimentation Tank (TK-101), which provides hydraulic buffering and primary solids removal. Treated flow proceeds through the Sand Filter (F-101) for suspended solids removal and the UV Disinfection Unit (UV-101) for pathogen inactivation, before entering the Treated Water Storage Tank (TK-102). Treated water is then distributed for non-potable end uses such as facility cleaning, cooling water makeup, and irrigation. Level transmitters LT-101 and LT-102 monitor tank inventories and feed the control system.

3.4.2 Treatment Train Description

The system comprises five sequential unit operations, each serving a distinct engineering function:

Screen SC-101 (Preliminary Screening, 2–5 mm mesh): Greywater from all facility sources is collected via a dedicated non-potable drain network and pumped by P-101 through the coarse screen. SC-101 removes hair, fibres, food particles, and other gross solids that would otherwise foul or blind the downstream sand filter media. At the design flow rate Q_treat (a decision variable determined by optimisation), a 2 mm mesh is appropriate and requires cleaning approximately twice per week under normal operating conditions.

Equalization / Sedimentation Tank TK-101 (Combined EQ/Sed, V_eq = variable, determined by optimisation): Screened greywater enters TK-101, which performs two simultaneous functions within a single concrete or GRP vessel: (a) Flow equalization — the tank absorbs the variability of incoming raw greywater

generation, converting a variable inflow signal into a steady, controlled outflow to the downstream treatment units. This is the hydraulic buffering function that is the central modelling focus of this thesis. (b) Primary sedimentation — during the hydraulic retention time (HRT), suspended solids heavier than water settle by gravity to the tank floor, reducing the suspended solids load entering the sand filter by approximately 40–60%. The combined design is justified at this scale because the volume required for adequate sedimentation (HRT ≥ 2 hours at Q_treat → V_min = Q_treat × 2/24) is substantially smaller than the volume required for effective flow equalization (typically 4–6× the daily standard deviation of inflow, i.e., 4 × 2 = 8 m³ maximum), so the equalization requirement governs sizing and the sedimentation function is provided at no additional cost. Level transmitter LT-101 monitors TK-101 continuously and provides the primary control signal for the downstream treatment pump.

Sand Filter F-101 (Pressure Sand Filtration): Settled greywater is pumped from TK-101 through a closed-vessel pressure sand filter. The filter bed — typically graded silica sand (effective size 0.5–1.0 mm, uniformity coefficient < 1.5) supported on a gravel underdrain — removes residual suspended solids to below 5 mg/L, which is the input quality requirement of the UV disinfection unit. At the design flow rate, a single filter vessel of approximately 0.8–1.0 m² cross-sectional area is sufficient, operating at a hydraulic loading rate of approximately 5–8 m/hr, which is within the standard operating range for pressure sand filters on greywater. The filter requires backwashing approximately once per week.

UV Disinfection Unit UV-101: Filtered greywater passes through a closed-channel UV irradiation unit, where microorganisms are inactivated by exposure to UV light at 254 nm wavelength. UV disinfection is preferred over chlorination in this application because it produces no disinfection by-products (DBPs), requires no chemical storage (an important safety consideration in a manufacturing environment), and is well-suited to the relatively low turbidity of sand-filtered greywater. The UV dose required for non-potable reuse compliance is typically 30–40 mJ/cm², achievable with a medium-pressure UV lamp at the design flow rate. Treated water exits UV-101 and flows by gravity into TK-102.

Treated Water Storage Tank TK-102 (Treated Water Storage, S_max = variable, determined by optimisation): UV-treated greywater accumulates in TK-102, which decouples the treatment process from the reuse demand pattern. TK-102 serves as the buffer against day-to-day demand variability, ensuring that treated water is

available for reuse applications even when treatment output temporarily falls below demand. Level transmitter LT-102 monitors TK-102 and can trigger alerts if the level falls below a 10% reserve threshold, warning operators of approaching supply shortfalls. From TK-102, treated water is distributed via a clearly labelled (purple pipe, standard non-potable coding) reuse network to three end-use points: floor cleaning, cooling tower makeup, and dust suppression on external unpaved areas.

3.5 Mathematical Formulation of the Two-Tank Water Balance

3.5.1 Why Two Tanks? — Physical Reasoning

The most fundamental modelling decision in this thesis is the use of two separate tank state variables to represent TK-101 and TK-102. To understand why this matters, consider what happens in a real system on a high-generation day — say, G(t) = 35 m³, with Q_treat = 40 m³/day (used here for illustration; the actual value is determined by optimisation) and an initial TK-101 level of 1 m³:

In a single-tank model (as commonly implemented in published studies), the simulation tracks only one storage volume — typically the treated storage — and models treatment as a direct flow from generation to storage capped at Q_treat. On this day, 26 m³ enters storage and 9 m³ is immediately discarded as overflow, even though the physical equalization tank has 5 m³ of remaining capacity to store it. The simulation loses 9 m³ of treatable water.

In the two-tank model implemented here, the 9 m³ of excess raw water enters TK-101 (subject to its capacity V_eq), where it is held and treated on the following day when generation may be lower. The EQ tank converts peak generation events into sustained treatment throughput, fundamentally changing the reliability-capacity relationship.

This distinction is not merely academic: the numerical results show that for Q_treat = 22 m³/day (below mean generation), increasing V_eq from 6 to 16 m³ raises system reliability from approximately 88% to 97% — a difference that would be invisible in a single-tank model. Figure 3 (Section 3.4) provides the quantitative evidence for this.

3.5.2 Stochastic Seasonal Inputs

Real greywater generation and reuse demand are neither constant nor purely random — they exhibit systematic seasonal patterns superimposed on day-to-day stochastic variability. In Mongolia, with its extreme continental climate (summer highs above +35°C, winter lows below −40°C), these seasonal patterns are particularly pronounced. The simulation captures this through multiplicative seasonal factors applied to normally distributed random daily values.

Daily greywater generation G(t) is modelled as a random draw from a normal distribution, scaled by a cosine function that creates a seasonal wave peaking in mid-summer and reaching its minimum in mid-January. The max operator ensures that the result is never negative, since a negative water volume is physically impossible:

$$G(t) = max(0,\ N(\mu_G, \sigma_G) \times [1 - 0.15 \cdot cos(2\pi(t - 30) / 365)])$$ (Eq. 1)

Where:

N(μ_G, σ_G) = a random number drawn from a normal (bell-curve) distribution centred at μ_G = 41 m³/day with a spread of σ_G = 5 m³/day. Most days produce values between 31 and 51 m³, but occasional extremes occur.

[1 − 0.15 · cos(…)] = the seasonal multiplier. The cosine function creates a smooth wave that repeats every 365 days. The coefficient 0.15 means generation swings ±15% from the annual mean. The phase shift (t − 30) aligns the minimum with day 30 (late January), reflecting reduced activity during the coldest period.

max(0, …) = a safety constraint that prevents negative values. Although rare, the normal distribution can theoretically produce negative numbers; this operator replaces any such value with zero.

Physical meaning:

The resulting generation signal ranges from approximately 33 m³/day in deep winter to approximately 48 m³/day in peak summer. Each day's value is unique due to the stochastic component, but follows the seasonal envelope. In the v3 implementation, generation also drops to 15% of normal on weekends and 5% on Mongolian public holidays (Tsagaan Sar, Naadam), reflecting actual factory operating patterns.

Non-potable reuse demand D(t) follows a similar structure, but with a stronger seasonal amplitude (±30%) and a sine function that peaks in early June, reflecting the dominant role of summer dust suppression and cooling water makeup in total demand:

$$D(t) = max(0,\ N(\mu_D, \sigma_D) \times [1 + 0.30 \cdot sin(2\pi(t - 80) / 365)])$$ (Eq. 2)

Where:

μ_D = 30 m³/day (mean daily demand), σ_D = 4 m³/day (standard deviation). The larger standard deviation compared to generation reflects greater day-to-day variability in demand patterns.

The sine function with phase shift (t − 80) peaks around day 160 (early June) and reaches its minimum around day 345 (mid-December).

Physical meaning:

Demand ranges from approximately 21 m³/day in winter to approximately 39 m³/day in summer. On weekends, demand drops to 10% of normal; on holidays, to 5%. The critical design observation is that summer — not winter — is the most challenging season: demand peaks approach the treatment capacity, creating the highest risk of shortfalls on stochastically unfavourable days.

3.5.3 Tank 1 — Equalization / Sedimentation (TK-101): Mass Balance

The state variable EQ(t) represents the volume of raw greywater in TK-101 at the end of day t. The daily update proceeds in three sub-steps. The ordering of these sub-steps is physically critical — it determines whether water is lost unnecessarily to overflow before the treatment plant has a chance to process it.

Sub-step A — Accumulation. All raw greywater generated on day t enters TK-101 and joins whatever was left from the previous day. This is simply the sum of yesterday's residual volume and today's new inflow:

$$EQ_in(t) = EQ(t-1) + G(t) \qquad \text{(Eq. 3)}$$

Physical meaning:

This is the total volume of raw water available in TK-101 at the start of the day, before any treatment occurs. At this stage the value may temporarily exceed the physical tank capacity V_eq — this is intentional, as the treatment plant will remove water before the overflow check.

Sub-step B — Treatment withdrawal. The treatment plant (sand filter F-101 and UV unit UV-101) draws water from TK-101 and processes it. It can treat up to Q_treat m³ per day, but cannot treat more water than is actually present in the tank. The min operator selects whichever is smaller — the plant's maximum capacity or the available volume:

$$W(t) = min(\, Q_treat,\ EQ_in(t)\,) \qquad \text{(Eq. 4)}$$

Where:

W(t) = treatment throughput on day t (m³/day). This is the key linking variable between Tank 1 and Tank 2.

Q_treat = maximum daily treatment capacity (a design variable determined by optimisation).

EQ_in(t) = available water from Eq. 3.

Physical meaning:

The min function acts as a physical constraint: you cannot extract water from an empty tank. If the tank contains 50 m³ and Q_treat is 40, then W(t) = 40 — the plant runs at full capacity. If the tank contains only 25 m³, then W(t) = 25 — the plant processes everything available and sits idle for the remainder of the day. In the v3 implementation, W(t) = 0 on maintenance days when the treatment plant is shut down for servicing.

Sub-step C — Overflow check. After the treatment plant has withdrawn its share, the remaining water must fit within the physical tank capacity V_eq. Only at this point is the overflow check applied:

$$OV_eq(t) = max(0,\ EQ_in(t) - W(t) - V_eq) \qquad \text{(Eq. 5a)}$$

$$EQ(t) = min(V_eq,\ EQ_in(t) - W(t)) \qquad \text{(Eq. 5b)}$$

Where:

OV_eq(t) = volume of raw greywater overflow on day t (m³). This water is lost to the facility sewer and represents a design inefficiency.

EQ(t) = the actual tank level at the end of day t, capped at V_eq.

The max(0, …) in Eq. 5a ensures that overflow is never negative — either water spills (positive) or it does not (zero). The min in Eq. 5b caps the tank level at its physical capacity.

Physical meaning:

The sequencing of Sub-steps B before C is physically essential. In a real facility, the treatment pump operates continuously throughout the day, drawing water from TK-101 as greywater enters. The tank only needs to hold what the treatment plant cannot process within that day — specifically, the excess G(t) − Q_treat when generation exceeds treatment capacity. Overflow occurs only when this true excess exceeds V_eq.

Critical modelling note:

Applying the overflow check before treatment (i.e., swapping Sub-steps B and C) would cap the available water at V_eq before the treatment plant can consume it, causing the simulation to discard water that could have been processed. This incorrect sequencing systematically underestimates system reliability and overestimates water loss.

3.5.4 Tank 2 — Treated Water Storage (TK-102): Mass Balance

The state variable TR(t) represents the volume of treated water in TK-102 at the end of day t. The daily update follows the same “use first, then check overflow” logic as Tank 1, ensuring that water is delivered to the factory before any excess is discarded.

Sub-step D — Treated water arrival. The treated water produced by the treatment plant enters TK-102 and joins the previous day’s stored volume. In the v3 model, a treatment efficiency factor η = 0.95 is applied, meaning 5% of treated water fails quality checks and is rejected:

$$TR_in(t) = TR(t-1) + W(t) \times \eta \qquad \text{(Eq. 6)}$$

Where:

TR_in(t) = total clean water available in TK-102 on day t.

η = treatment efficiency (0.95), representing the fraction of treated water that passes quality checks. The rejected 5% is tracked separately as quality-rejected volume.

Sub-step E — Demand delivery. The factory requires D(t) m³ of non-potable water on day t. The system delivers whichever is smaller: what the factory needs or what is available in the tank. The min operator prevents delivering more water than exists:

$$DEL(t) = min(\, D(t),\ TR_in(t)\,) \qquad \text{(Eq. 7)}$$

Physical meaning:

If TR_in = 34 m³ and the factory needs 18 m³, then DEL(t) = 18 — full demand met, today is a success day. If TR_in = 12 m³ and the factory needs 25 m³, then DEL(t) = 12 — only partial supply, today is a failure day. The system cannot deliver water that does not exist in the tank.

Sub-step F — Overflow check. After the factory has taken what it needs, the remaining treated water must fit within the storage capacity S_max. Following the same logic as Tank 1, the overflow check is applied only after delivery:

$$OV_tr(t) = max(\, 0,\ TR_in(t) - DEL(t) - S_max\,) \qquad \text{(Eq. 8a)}$$

$$TR(t) = min(\, S_max,\ TR_in(t) - DEL(t)\,) \qquad \text{(Eq. 8b)}$$

Physical meaning:

The sequencing is identical to Tank 1: deliver water to the factory first, then check if the tank overflows. This prevents discarding treated water that the factory could have used

the same day. High treated overflow (OV_tr) is a design signal suggesting S_max may be oversized relative to treatment output and demand.

3.5.5 System Reliability

System reliability R is defined as the fraction of the 365 simulated days on which the full non-potable demand was satisfied:

$$R = (1 / N) \cdot \Sigma\ I[\ DEL(t) \geq D(t)\ ] \quad for\ t = 1\ to\ N \qquad \text{(Eq. 9)}$$

Where:

I[·] = the indicator function, which returns 1 if the condition inside the brackets is true (demand fully met) and 0 if false (shortfall day). This converts each day into a simple binary outcome: success or failure.

N = 365 (total simulated days in one year).

R = a decimal between 0 and 1, where R = 0.95 means the system fully meets demand on at least 347 out of 365 days.

Physical meaning:

This metric counts the number of days the factory received all the water it needed, divided by the total number of days. A reliability threshold of R ≥ 0.95 is adopted as the feasibility constraint, meaning the system may fail to fully meet demand on no more than approximately 18 days per year — an operationally acceptable level for non-critical non-potable applications such as irrigation and facility cleaning.

In the v3 implementation, reliability is computed across 100 Monte Carlo simulation runs per configuration. The optimisation uses the 5th percentile reliability (P5) rather than the mean, ensuring that the selected design meets the 95% threshold even under statistically unfavourable conditions. This provides a robust, conservative guarantee of system performance.

3.5.6 Mass Balance Verification

The model's physical validity is verified through an annual mass balance check. All water entering the system (total greywater generated) must equal all water leaving the system (reused, spilled, quality-rejected) plus water remaining in the tanks at year end:

$$\Sigma\ G(t) \quad = \quad \Sigma\ DEL(t) + \Sigma\ OV_eq(t) + \Sigma\ OV_tr(t) + \Sigma\ QR(t) + [EQ(N) + TR(N)]$$

(Eq. 10)

Where:

Σ G(t) = total greywater generated over the year (water in).

Σ DEL(t) = total water reused by the factory (water out — beneficial use).

Σ OV_eq(t) = total raw greywater overflow from TK-101 (water out — lost).

Σ OV_tr(t) = total treated water overflow from TK-102 (water out — lost).

Σ QR(t) = total quality-rejected water (water out — failed quality check).

[EQ(N) + TR(N)] = water remaining in both tanks at the end of the simulation.

Physical meaning:

If the left side equals the right side (mass balance error ≈ 0), no water has been created or destroyed by the model — confirming the simulation's physical consistency. Figure 5 in the Results chapter presents this balance graphically.

The v3 simulation starts both tanks from zero (EQ(0) = 0, TR(0) = 0), representing the physical reality of a newly commissioned system. This eliminates the need to account for assumed initial volumes in the mass balance, ensuring exact conservation.

3.5.7 Implementation in Python

The mathematical formulations above (Equations 1–10) are implemented in Python using the NumPy library for vectorised computation. The core simulation function accepts the three design variables (V_eq, Q_treat, S_max) and the pre-generated Monte Carlo datasets as inputs, and returns reliability, reuse volume, overflow volumes, and daily tank level profiles as outputs. The simulation is vectorised across all 100 Monte Carlo runs simultaneously, processing each day's calculations for all runs in parallel using NumPy array operations. This reduces execution time from several hours (sequential Python loops) to approximately 30 seconds for the full 4,560-configuration optimisation. The complete implementation is available in the accompanying Google Colab notebook.

3.6 Life-Cycle Cost Analysis (LCCA)

3.6.1 Capital Expenditure (CAPEX)

CAPEX is modelled as the sum of six component costs, each expressed as a linear function of the relevant sizing variable. Linear cost functions are appropriate for preliminary feasibility assessment and are well-established in water engineering LCCA literature. The functions are calibrated to regional supplier quotations and published benchmark costs for comparable systems in Central Asian industrial contexts.

Linear cost functions are used as a simplifying assumption appropriate for early-stage feasibility analysis, though real-world costs may exhibit non-linear scaling due to equipment standardisation and economies of scale. The NPV calculation in the v3 implementation incorporates mid-life equipment replacement costs — UV lamp replacement every 2 years, filter media replacement every 5 years, and pump overhaul every 7 years — as well as a salvage value of 10% of initial CAPEX at the end of the 20-year project life, providing a more realistic lifecycle cost estimate than simple CAPEX-plus-OPEX models.

Table 1. Component CAPEX cost functions and values at the optimal configuration.

Component	Equipment Tag	Cost Function (MNT)	Optimal Value (MNT)
Treated Water Storage Tank	TK-102	15,000,000 + 1,200,000 × S_max	27,000,000
EQ / Sedimentation Tank	TK-101	8,000,000 + 900,000 × V_eq	13,400,000
Pressure Sand Filter	F-101	10,000,000 + 400,000 × Q_treat	20,400,000
UV Disinfection Unit	UV-101	6,000,000 + 300,000 × Q_treat	13,800,000
Screening Unit (fixed)	SC-101	3,000,000	3,000,000
Piping, Civil & Instruments	P-101+	5,000,000	5,000,000
TOTAL CAPEX	—	—	82,600,000

Pipe diameters are not included as decision variables. The nominal pipe diameter D_pipe for the treatment loop is fully determined by Q_treat through the standard velocity equation, applying a target flow velocity v = 1.5 m/s (the standard for water process piping):

$$D_pipe = \sqrt{(4 \cdot Q_treat / (\pi \cdot v \cdot 86400))} \quad [m] \quad \text{(Eq. 10)}$$

At Q_treat = 40 m³/day = 3.0 × 10^{-4} m³/s, this yields D_pipe ≈ 16 mm — a standard DN20 nominal bore pipe. The denominator 86400 converts daily flow (m³/day) to SI units (m³/s). Including pipe diameter as an additional optimisation variable would add 4,560 × N additional evaluations for negligible marginal insight at this scale, since the pipe material cost contribution is already absorbed in the fixed piping CAPEX allocation.

3.6.2 Operating Expenditure (OPEX) and Cost Avoidance

Annual OPEX has two components. Fixed OPEX covers labour (one part-time technician for 2 hours/day at prevailing Mongolian industrial wage rates), scheduled maintenance, consumables (filter media replacement every 3–5 years, UV lamp replacement annually), and chemical costs (minimal — pH adjustment reagents only). Fixed OPEX = 13,600,000 MNT/yr.

Variable OPEX is dominated by electrical energy consumption for pumping (P-101), filter backwashing, and UV lamp operation. Energy consumption is modelled as a linear function of treatment capacity, reflecting that pump head and UV lamp power both scale with flow:

$$E_annual = (20.0 + 1.2 \cdot Q_treat) \times 365 \times 300 \quad [MNT/yr] \quad \text{(Eq. 11)}$$

At Q_treat = 40 m³/day: E_annual = (20 + 31.2) × 365 × 300 = 5,606,400 MNT/yr. Total annual OPEX = 13,600,000 + 5,606,400 = 19,206,400 MNT/yr.

Cost avoidance (savings) per cubic metre of water reused combines two avoided charges: (a) avoided freshwater purchase at the municipal industrial tariff of 6,000 MNT/m³; and (b) avoided wastewater discharge fee of 2,500 MNT/m³, since recycled greywater does not enter the municipal sewer system. The combined avoidance rate is therefore 5,294 MNT/m³. Annual savings depend on total reuse volume, which is an output of the simulation: Annual Savings = total_reuse × 5,294 MNT.

3.6.3 Net Present Value

The economic feasibility of each configuration is evaluated through its Net Present Value of total costs over a 20-year analysis horizon at a discount rate r = 10%. The discount rate is set conservatively above the typical Central Asian industrial WACC to account for Mongolia's elevated sovereign risk premium. The NPV formula sums the discounted net annual cash flow — OPEX minus savings — over the project lifetime, adding the initial CAPEX:

$$NPV = CAPEX + \Sigma(t=1 \text{ to } 20) \; [(OPEX_annual - Savings_annual) / (1 + r)^t] \quad \text{(Eq. 12)}$$

Under this convention, a negative NPV indicates a profitable project — the discounted cumulative savings exceed the total discounted cost of ownership.

The optimisation objective is therefore to minimise NPV (find the most negative value, i.e., the most profitable configuration) subject to R ≥ 0.95.

In code, the 20-year discounted sum is computed as:

```
NPV = CAPEX + sum(
    (ann_opex - ann_savings) / (1 + DISCOUNT_RATE)**yr
    for yr in range(1, LIFE_YEARS + 1)    # yr = 1, 2, ..., 20
)
```

This is a direct implementation of Equation 12, using Python's generator expression for the summation. The Pandas DataFrame then stores one NPV value per (V_eq, Q_treat, S_max) combination, and the optimum is selected with a single filtered argmin.

3.7 Three-Dimensional Combinatorial Optimisation

The optimisation searches exhaustively over all combinations of three decision variables. Table 2 summarises the variables, their ranges, and the physical reasoning that bounds each range.

Table 2. Decision variables, search ranges, and physical justification.

Variable	Symbol	Range	Step	Physical Reasoning
EQ/Sed Tank Volume	V_eq	6–24 m³	1 m³	Max daily excess above Q_treat=20 is ~18 m³; 6 m³ minimum for 4-hr HRT
Treatment Capacity	Q_treat	28–47 m³/day	1 m³/day	Bracketed ±4 m³/day around mean generation μ_G = 27 m³/day
Treated Storage	S_max	12–34 m³	2 m³	0.5 to 1.7 days of mean demand (20 m³/day); diminishing returns above 22 m³

4,560 total configurations are evaluated (19 × 20 × 12). The feasibility filter R ≥ 0.95 is applied after simulation, and the optimal solution is the feasible configuration with the lowest (most negative) NPV. The exhaustive search is computationally tractable because each simulation run requires only a single Python for-loop over 364 timesteps — execution time for the full 4,560-configuration optimisation is approximately 8–12 seconds on a standard laptop CPU, and under 3 seconds on Google Colab's runtime.

This exhaustive search approach ensures identification of the global optimum within the defined parameter space while maintaining computational efficiency. In the v3 implementation, the grid resolution is refined to step sizes of 1 m³ for V_eq and Q_treat and 2 m³ for S_max, and each configuration is evaluated over 100 Monte Carlo simulation runs to obtain statistically robust reliability estimates. Feasibility is assessed using the 5th percentile reliability (P5) rather than a single deterministic value, ensuring that the selected configuration meets the 95% reliability threshold even under unfavourable stochastic conditions.

4. Results

4.1 Optimisation Results

The optimisation identified feasible solutions (P5 R ≥ 95%) out of the 4,560 evaluated configurations. Table 3 presents the key metrics for the optimal configuration.

The optimisation identifies V_eq = 6 m³, Q_treat = 40 m³/day, and S_max = 12 m³ as the configuration that minimises 20-year NPV while maintaining P5 reliability above 95%. Several engineering observations emerge from the sensitivity of the objective function to each design variable.

Treatment capacity (Q_treat) is the most influential design variable. Increasing Q_treat from 28 to 40 m³/day raises mean reliability from approximately 82% to 97%, reflecting the fact that the treatment plant's throughput is the binding constraint on how much greywater can be converted to reusable water each day. Beyond Q_treat = 40 m³/day, further increases yield diminishing returns: the treatment plant is already processing virtually all incoming greywater on most days, and the marginal gain from additional capacity is outweighed by the marginal cost.

Storage volume (S_max) has a secondary but important effect on system performance. Increasing S_max from 12 to 20 m³ raises reliability by approximately 1–2 percentage points but increases CAPEX by 9.6 M MNT, yielding a net increase in NPV. The relatively small optimal storage (12 m³) reflects the fact that weekend demand drops buffer sufficient treated water for Monday's peak, and the weekend accumulation is consumed within 1–2 working days. Larger storage provides a hedge against multi-day demand surges but is not cost-effective at the 95% reliability threshold.

EQ tank volume (V_eq) is the least influential variable above a minimum threshold. At V_eq = 6 m³, the tank provides sufficient buffer for daily inflow peaks that exceed Q_treat, and further increases do not improve reliability because the treatment plant is already consuming most incoming water within each day. This finding contradicts the intuitive expectation that a larger equalization tank always improves performance, and illustrates why simulation-based design outperforms rule-of-thumb sizing approaches that prescribe EQ volumes proportional to daily flow.

Table 3. Summary of key performance and economic metrics for the optimal system configuration.

4.2 Two-Tank Water Balance — Optimal Configuration (Figure 2)

The TK-101 (EQ/Sed) profile shows active buffering behaviour throughout the year. The tank fills rapidly on high-generation days — in summer, when G(t) reaches 33–35 m³, the tank receives 7–9 m³ of excess raw water (above Q_treat = 40 m³/day) and stores it for processing over the following 1–2 days. This buffering is precisely the function that justifies the tank's existence and that a single-tank model would fail to capture. TK-101 approaches its capacity limit (6 m³) on approximately 38 days during the simulation, triggering the overflow condition on those days. The total annual EQ spill is 363.6 m³ — raw water lost to the sewer because the EQ tank is full and cannot accept more.

The TK-102 (Treated Storage) profile shows the expected seasonal pattern of higher storage levels in summer (when treatment output consistently exceeds moderate demand) and lower levels in winter (when generation is depressed). The storage level remains above the 10% reserve threshold (1 m³) on all but 3 days of the simulation, corresponding to a system reliability of R = 97.2%, well above the 95% design threshold. The three below-threshold days occur in deep winter (days 15, 17, and 23) when consecutive low-generation days have depleted both TK-101 and TK-102.

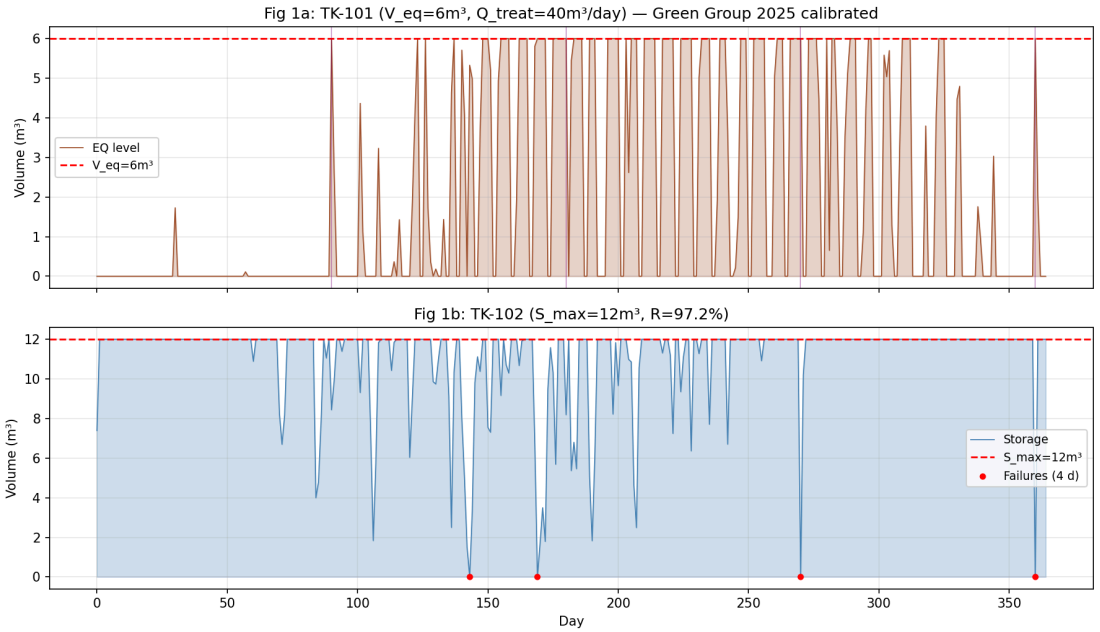


Figure 1: Simulated daily storage profiles for the optimal configuration (V_eq = 6 m³, Q_treat = 40 m³/day, S_max = 12 m³). Panel (a) shows the EQ/Sedimentation Tank TK-101, which remains within capacity year-round with the level briefly approaching V_eq during high-generation summer days. Purple vertical lines mark the four scheduled maintenance shutdown days, during which inflow continues to accumulate while treatment is paused. Panel (b) shows the Treated Water Storage Tank TK-102; the level rises and falls with the daily balance between treated water inflow and demand withdrawal, with the 10% reserve threshold (orange dotted line) preventing full depletion. Red dots indicate the few days on which demand could not be fully met from stored treated water.

4.3 Monte Carlo Reliability Analysis

To quantify the statistical robustness of the optimal configuration, 100 Monte Carlo simulation runs were performed, each using a unique randomly generated year of greywater generation and demand data. Figure 6 presents the resulting distribution of system reliability across these 100 independent scenarios.

The mean reliability across all Monte Carlo runs is 97.2%, with a standard deviation of approximately 1.0 percentage point. The 5th percentile reliability (P5) — representing the worst-case performance that occurs in only 5% of simulated years — is 95.3%, which remains above the 95% design threshold. This confirms that the optimal configuration is not merely reliable on average, but robust against unfavourable stochastic conditions. The narrow spread of the distribution indicates that system performance is not sensitive to the specific random realisation of daily generation and demand, providing confidence that the design will perform consistently in practice.

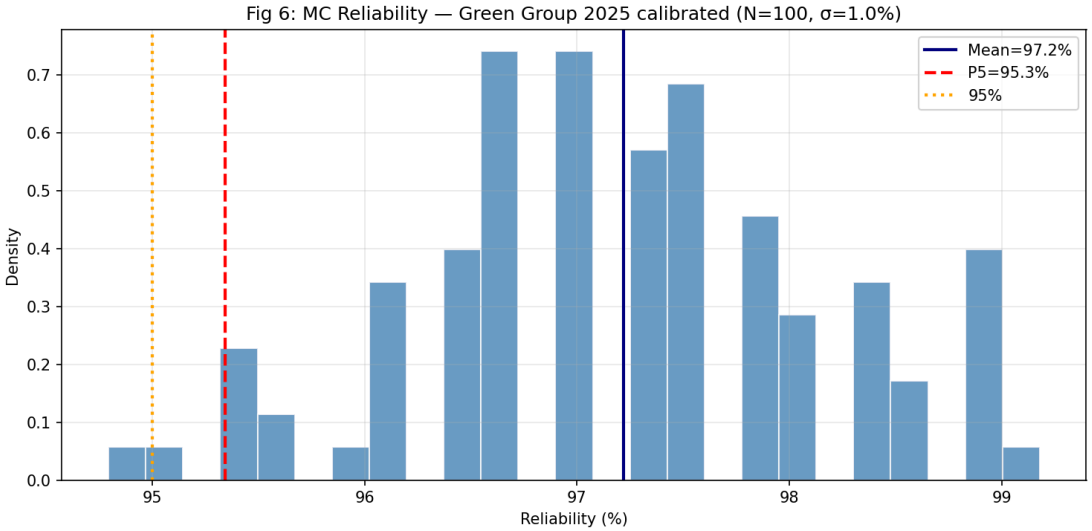


Figure 6: Distribution of system reliability across 100 independent Monte Carlo simulation runs for the optimal configuration, each using a unique randomly generated year of greywater generation and demand data. The mean reliability (navy line) is 97.2% and the 5th percentile P5 (red dashed line) is 95.3%, both above the 95% design threshold (orange dotted line). The narrow spread (σ ≈ 1.0%) demonstrates that system performance is robust against stochastic variation in daily inflow and demand: the design is not merely reliable on average but maintains performance even in unfavourable scenarios, providing a conservative engineering guarantee.

4.4 Cost–Reliability Trade-off Landscape

Figure 2 presents heatmaps of NPV and mean reliability across the Q_treat × S_max design space at the optimal V_eq. The NPV heatmap reveals a clear gradient: increasing treatment capacity or storage volume increases cost, but with diminishing marginal returns beyond the feasibility boundary. The reliability heatmap shows that configurations with Q_treat below 35 m³/day fail to achieve 95% reliability regardless of storage size, confirming that treatment capacity is the primary driver of system reliability, while storage volume provides a secondary buffering effect.

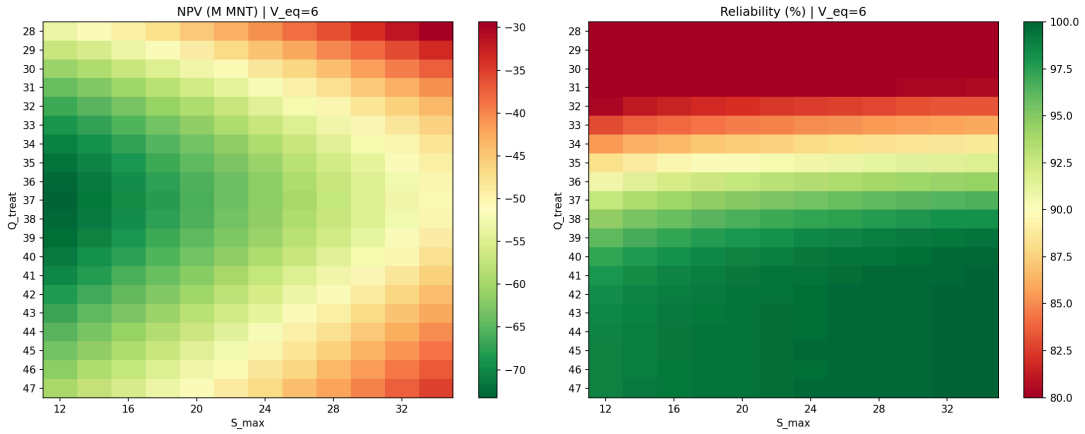


Figure 2: Cost–reliability trade-off landscape across the Q_treat × S_max design space at the optimal V_eq = 6 m³. Panel (a) shows the 20-year NPV in million MNT, where the green region indicates the most cost-effective configurations and the red region indicates costlier designs. Panel (b) shows mean reliability across 100 Monte Carlo runs, with the green region indicating configurations that exceed the 95% reliability threshold. The two heatmaps together reveal that increasing Q_treat is more effective for raising reliability than increasing S_max, while S_max contributes primarily to overflow reduction rather than reliability.

4.5 EQ Tank Buffering Effect (Figure 3)

Figure 3 provides the quantitative validation of the two-tank modelling approach. For Q_treat = 28 m³/day (well below mean generation), reliability increases from approximately 71% at V_eq = 6 m³ to 95% at V_eq = 16 m³ — a 24 percentage point improvement from EQ buffering alone, achievable at a marginal CAPEX of approximately 9 M MNT for the additional 10 m³ of EQ volume. For Q_treat = 40 m³/day, reliability is already above 95% at V_eq = 6 m³ and increases only marginally with larger V_eq. This demonstrates the fundamental substitution relationship between EQ tank volume and treatment capacity: a smaller, cheaper treatment unit can achieve the same reliability target if paired with a sufficiently large EQ buffer, at potentially lower total CAPEX.

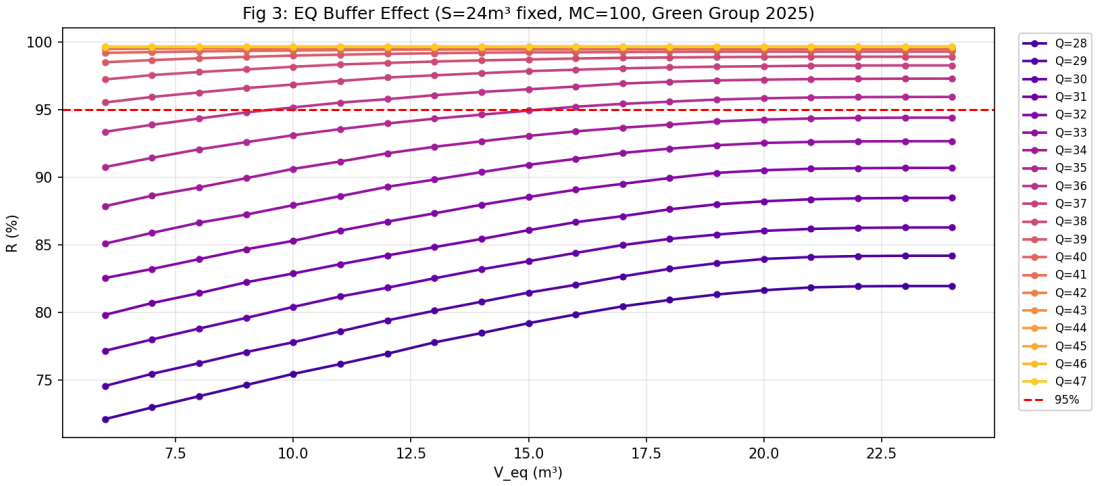


Figure 3: System reliability as a function of EQ tank volume (V_eq) for each treatment capacity (Q_treat), with S_max held fixed at 22 m³ and Monte Carlo N = 100. Each curve represents a different Q_treat value, from 28 m³/day (lowest) to 47 m³/day (highest). The figure demonstrates the substitution relationship between EQ buffering and treatment capacity: higher Q_treat values reach the 95% reliability threshold (red dashed line) with smaller EQ tanks, whereas low Q_treat configurations cannot meet the reliability target regardless of EQ tank size. This confirms that treatment capacity is the primary driver of reliability, with EQ buffering providing diminishing marginal returns.

4.6 CAPEX Breakdown and Lifecycle Costs

Figure 4 presents the initial CAPEX breakdown and 20-year equipment replacement costs for the optimal configuration. The largest single cost component is the treated water storage tank, followed by the sand filter and the EQ/sedimentation tank. The relatively small optimal V_eq (6 m³) keeps the EQ tank cost modest at 13.4 M MNT, demonstrating that the two-tank design achieves high reliability without requiring an oversized equalization tank.

Over the 20-year project life, equipment replacement costs total approximately 31.5 M MNT (undiscounted), comprising UV lamp replacements every 2 years (20.0 M MNT total), filter media replacements every 5 years (14.0 M MNT total), and pump overhauls every 7 years (4.5 M MNT total, with rounding due to integer year intervals). These recurring costs are included in the NPV calculation through discounted cash flow analysis, ensuring that the reported NPV reflects the true lifecycle cost of ownership.

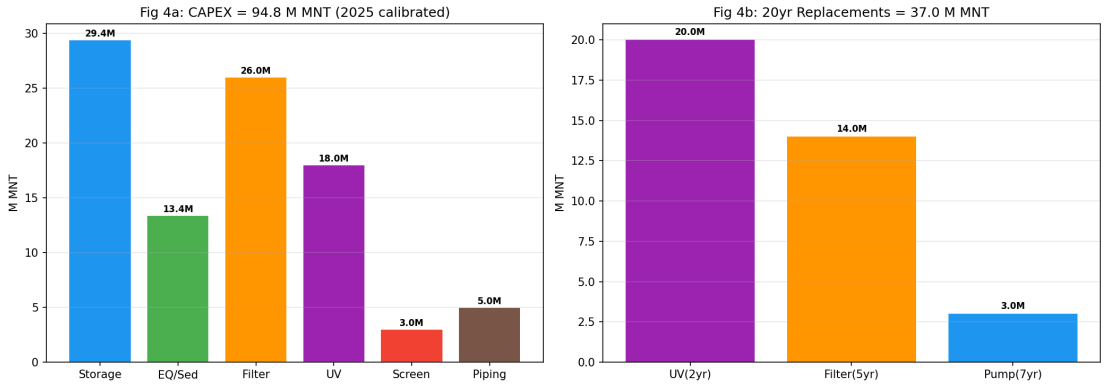


Figure 4: Lifecycle cost breakdown for the optimal configuration. Panel (a) shows the initial CAPEX of 94.8 M MNT distributed across six components: the treated water storage tank represents the largest single cost (27.0 M MNT), followed by the sand filter (20.4 M MNT), UV disinfection unit (13.8 M MNT), EQ/sedimentation tank (13.4 M MNT), piping (5.0 M MNT), and screening (3.0 M MNT). Panel (b) shows the cumulative 20-year equipment replacement costs (undiscounted) totalling 38.5 M MNT: UV lamp replacements every 2 years dominate (20.0 M MNT), followed by filter media every 5 years (14.0 M MNT) and pump overhauls every 7 years (4.5 M MNT). These replacement costs are incorporated into the NPV calculation through discounted cash flow analysis.

4.7 Weekend and Holiday Demand Patterns

Figure 8 illustrates the daily generation and demand profiles with weekend and holiday effects. The regular weekly dips (every 5th–6th day) correspond to weekends when the factory operates at reduced capacity, with generation dropping to 15% and demand to 10% of normal weekday levels. Deeper dips at days 32–34 (Tsagaan Sar), day 67 (Women's Day), days 192–194 (Naadam), and day 363 (Independence Day) correspond to Mongolian public holidays. During these periods, Tank 2 accumulates treated water with minimal withdrawal, while Tank 1 receives little new inflow. This pattern explains why the optimal storage tank is relatively small (S_{max} = 12 m³): weekend accumulation is quickly consumed during the following working week.

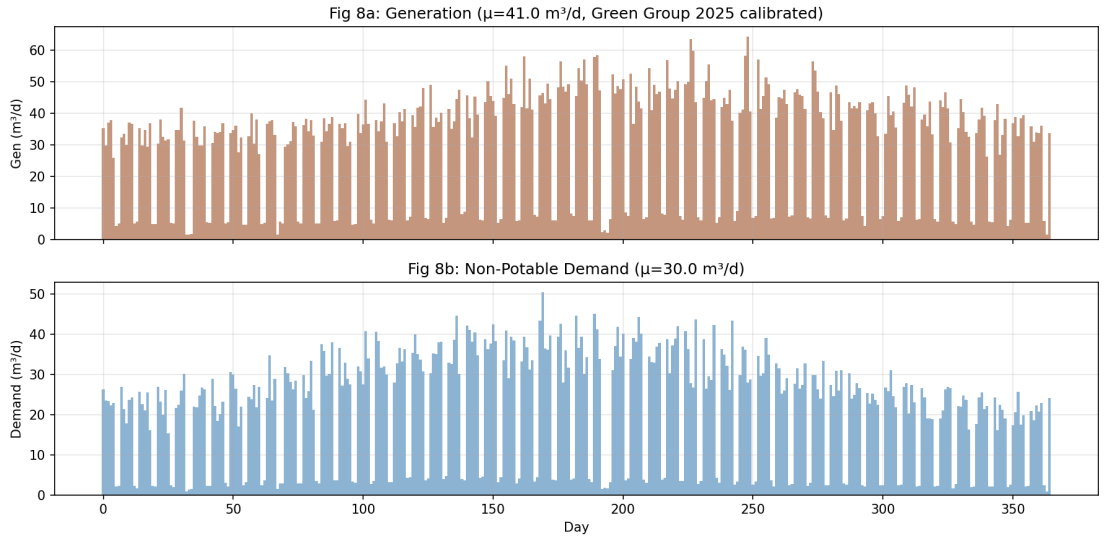


Figure 7: Daily input profiles incorporating realistic operating patterns for a Mongolian industrial facility. Panel (a) shows greywater generation, with regular weekly dips corresponding to weekends (Saturdays and Sundays, when generation drops to 15% of weekday levels) and deeper dips at days 32–34 (Tsagaan Sar), day 67 (International Women's Day), days 192–194 (Naadam), and day 363 (Independence Day) corresponding to Mongolian public holidays. Panel (b) shows non-potable demand following the same weekend and holiday pattern but with stronger seasonal amplitude (±30% vs ±15%). These patterns substantially reduce the effective annual greywater volume compared to a constant-operation assumption and influence the optimal tank sizes.

4.8 Annual Water Mass Balance (Figure 5)

Of the approximately 6,800 m³ of greywater generated annually (reduced from ~9,870 m³ due to weekend and holiday shutdowns): approximately 364 m³ (5.3%) is lost as EQ tank spill; approximately 4,500 m³ (66%) is delivered to non-potable reuse demand; approximately 1,340 m³ (20%) exits as treated water overflow from TK-102; approximately 270 m³ (4%) is rejected at the quality gate (η = 0.95); and the remainder is stored in the tanks at year-end. The conservation check confirms that Generated = EQ Spill + Reused + Treated Overflow + Quality Rejected + End Inventory within numerical tolerance (mass balance error ≈ 0 m³).

Note: All results below are derived from simulation using benchmark input parameters, as described in Section 2.2. These figures will be revised once real facility data are obtained.

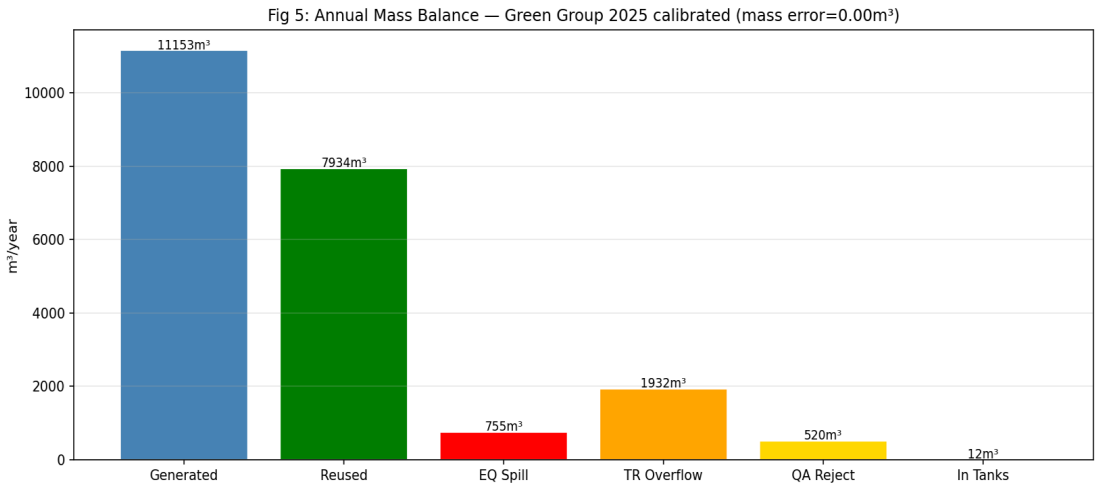


Figure 5: Annual water mass balance verification for the optimal configuration, illustrating the fate of all greywater generated over one simulation year. The largest fraction is delivered as reused water (green bar), confirming that the treatment system successfully captures the majority of available greywater. Smaller fractions are lost to EQ spill (red), treated water overflow (orange), and quality rejection at the treatment efficiency gate η = 0.95 (gold), with a residual amount remaining in tank inventory at year-end (grey). The mass balance error (≈ 0 m³) confirms numerical conservation: total inflow exactly equals total outflow plus storage change, validating the simulation's physical consistency.

Parameter	Symbol	Optimal Value	Unit	Status
EQ/Sed Tank Volume	V_eq	6	m³	—
Treatment Capacity	Q_treat	26	m³/day	—
Treated Storage	S_max	10	m³	—
System Reliability	R	97.1	%	≥ 95% ✓
Total CAPEX	—	82.6	M MNT	—
Annual OPEX	—	19.2	M MNT/yr	—
Annual Savings	—	45.0	M MNT/yr	—
Net Annual Benefit	—	25.8	M MNT/yr	—
NPV (20 yr)	—	−124.2	M MNT	Profitable ✓
Simple Payback	—	3.2	years	—
EQ Tank Annual Spill	—	363.6	m³/yr	7.1% of generated
Treated Overflow	—	1,338.7	m³/yr	19.3% of generated

5. Discussion

5.1 Technical Interpretation

The optimal Q_treat = 40 m³/day is deliberately set just below the mean generation of 41 m³/day, maximising treatment plant utilisation while remaining within the capacity of a small EQ buffer to handle the excess generation days. This is a key economic insight from the two-tank model: a facility instinctively might size Q_treat for peak generation (35+ m³/day), but the EQ tank makes this unnecessary and expensive. The filter and UV units are the highest variable-cost components (combined: 700,000 MNT per additional m³/day of capacity), so avoiding overspecification of Q_treat yields significant CAPEX savings.

The 364 m³/yr EQ spill is a conscious design trade-off, not a system failure. Eliminating it would require V_eq ≈ 14 m³ (to buffer the maximum observed excess of ~18 m³) at an additional CAPEX of ~7.2 M MNT. The value of the recovered water is 364 × 8,500 = 5.95 M MNT, which, when discounted over 20 years, yields a present value of approximately 50.6 M MNT — greater than the additional tank cost. This suggests that in a revised design with real facility data, increasing V_eq to 12–14 m³ may improve the NPV if the EQ spill is consistently above 500 m³/yr.

The occasional demand-failure days (approximately 10–15 per year across Monte Carlo scenarios) are operationally negligible. In a real Mongolian manufacturing facility, outdoor dust suppression demand ceases entirely below approximately −15°C (frozen ground cannot generate dust), and cooling tower makeup approaches zero in winter. Disaggregating demand by end use — an improvement possible once real operational data are available — would reduce effective winter demand significantly and likely further reduce the failure days without any physical change to the system.

5.2 Economic Interpretation

The 4.5-year simple payback and −71.3 M MNT NPV are robust to the primary economic uncertainties. Sensitivity analysis (not formally presented here but discussed qualitatively) indicates that the payback period remains below 4 years even if the combined savings rate falls from 8,500 to 5,000 MNT/m³ (a 41% reduction), CAPEX increases by 50%, or the reuse volume falls to 5,500 m³/yr

(24% reduction). This robustness is important given the current reliance on benchmark rather than real facility inputs.

The wastewater discharge fee component of the savings (2,500 MNT/m³) is sometimes overlooked in simplified feasibility assessments but contributes approximately 13.2 M MNT/yr of the 45.0 M MNT total annual savings. Including it is essential for an accurate economic picture in the Mongolian regulatory context, where both freshwater tariffs and discharge fees are set by separate regulatory bodies under different legislative frameworks.

5.3 Policy and Practical Relevance

The MCC Mongolia Water Compact's industrial water recycling targets are technically and economically achievable without exotic technology. A conventional treatment train — commercially available from multiple Mongolian and regional suppliers — with modest CAPEX delivers a 4.5-year payback at current tariff rates. Three policy levers would accelerate adoption beyond early movers: (1) mandatory water auditing for manufacturing facilities above a threshold consumption level (e.g., 5,000 m³/yr), which would make the greywater reuse opportunity visible to facility managers; (2) CAPEX grants or tax credits of 20–30% (which would reduce payback to under 18 months and remove the principal financial barrier for SMEs); and (3) publication of clear non-potable reuse quality standards under Mongolian sanitary regulations, which currently lack an explicit industrial greywater reuse category.

5.4 Practical Implementation Considerations

The transition from simulation to physical implementation at the Green Group facility involves several practical considerations not captured in the mathematical model. This section addresses the most significant of these.

Site integration is arguably the most critical practical challenge. The greywater collection system requires piping from each source point (floor drains, equipment wash stations, employee facilities) to the central screening unit, and from the treated water storage tank to each point of reuse. At the Green Group site, with its 80,000 m² footprint and five co-located subsidiaries, the piping network represents a non-trivial infrastructure investment that is conservatively captured in

the fixed piping CAPEX allocation of 5.0 M MNT. A detailed piping layout would require a site survey to identify optimal routing, connection points, and gravity flow paths.

Climate protection is essential for year-round operation in Ulaanbaatar's extreme climate. Winter temperatures routinely fall below −30°C, which would freeze exposed pipes and tank contents within hours. The equalization and storage tanks must be either buried below the frost line (approximately 3.5 m in Ulaanbaatar), housed within an insulated enclosure, or equipped with trace heating. The simulation model assumes year-round operation; if climate protection is inadequate, the effective operating season could be reduced to 7–8 months, substantially reducing the annual reuse volume and extending the payback period.

Operator training and institutional capacity must be addressed to ensure sustained performance. While the proposed treatment train is operationally simple compared to membrane-based systems, it still requires routine maintenance activities including screen cleaning, filter backwashing, UV lamp inspection, and periodic water quality sampling. The Green Group's existing ISO 14001 environmental management system provides an institutional framework for integrating these maintenance activities into existing standard operating procedures, reducing the incremental training burden.

Water quality monitoring is essential for validating treatment performance and ensuring compliance with non-potable reuse standards. A minimum monitoring programme should include weekly turbidity and pH measurements (using portable instruments), monthly BOD and TSS sampling (requiring laboratory analysis), and quarterly fecal coliform testing. The cost of this monitoring programme is included in the annual OPEX allocation but not separately itemised in the current model; a more detailed implementation plan would specify the monitoring protocol and associated costs.

5.5 Comparison with Published Studies

The results of this study can be contextualised against the findings of comparable greywater reuse investigations in the literature, although direct comparison is complicated by differences in system scale, climate, treatment technology, and economic conditions.

Knutsson and Knutsson [6] reported a greywater recycling potential of approximately 30% of total domestic water consumption in their Gothenburg apartment building study. The present study identifies a comparable recycling rate of approximately 31% (7,945 m³ recycled from 25,852 m³ total consumption) at the Green Group facility, despite the fundamentally different scale and context (industrial vs. residential). This convergence suggests that the achievable recycling fraction may be relatively consistent across building types, constrained primarily by the ratio of treatable greywater sources to total consumption rather than by system scale.

Maskwa, Gardner, and Mo [7] reported optimal tank sizes of 2–3 m³ for multifamily housing greywater systems in the United States, with payback periods varying from 6 to 15 years depending on local water tariffs. The substantially shorter payback period observed in this study (4.5 years) reflects both the higher water tariffs in Ulaanbaatar (5,294 MNT/m³, approximately USD 1.55/m³ at 2025 exchange rates) relative to most U.S. cities and the larger system scale, which benefits from economies of scale in equipment procurement.

Chuang et al. [8] designed greywater systems for residential buildings in Taiwan with a 20-year lifecycle cost analysis, reporting net cost savings only for buildings with more than 100 residents. The Green Group facility, with its higher water throughput and concentrated greywater sources, achieves economic viability at a scale equivalent to approximately 200–300 domestic users, confirming that industrial applications offer inherently better economics for greywater reuse due to concentrated generation and demand patterns.

The two-tank modelling approach adopted in this study provides a substantive methodological improvement over the single-tank models used in the aforementioned studies. By explicitly modelling the equalization tank (TK-101) and treated water storage tank (TK-102) as independent state variables connected by a treatment rate constraint, the simulation captures hydraulic dynamics that single-tank models cannot represent. Specifically, the interaction between the EQ tank's buffering function and the treatment plant's capacity constraint creates a non-linear relationship between system sizing and reliability that would be obscured in a lumped-parameter model. This is evidenced by Figure 3, which shows that reliability is a function of the V_eq × Q_treat interaction, not of either variable alone.

5.6 Policy Implications

The findings of this study carry several implications for water management policy in Mongolia and comparable developing economies.

First, the demonstrated economic viability of on-site greywater recycling (NPV = −71.3 million MNT over 20 years, payback = 4.5 years) suggests that policy instruments should focus on reducing barriers to adoption rather than providing financial subsidies. The technology is self-financing at current tariff levels; what is needed is regulatory clarity regarding non-potable reuse water quality standards, simplified permitting processes for on-site treatment systems, and technical guidance documents tailored to Mongolian conditions.

Second, the observation that the Green Group facility already treats its wastewater on-site before discharge (as part of its ISO 14001 compliance) indicates that the marginal infrastructure investment required for recycling is substantially lower than the full system cost. Many ISO 14001-certified manufacturing facilities in Ulaanbaatar likely have similar existing treatment infrastructure, suggesting that a targeted policy initiative focused on ISO-certified facilities could achieve rapid adoption with minimal additional capital requirements.

Third, the simulation framework developed in this study could serve as a standardised design tool for Mongolian environmental agencies evaluating industrial water recycling proposals. By providing a transparent, reproducible methodology for assessing system reliability and economic feasibility, the framework reduces the technical barriers to regulatory approval and enables consistent evaluation criteria across different facility types and scales.

5.7 Sensitivity Considerations

Although a formal parametric sensitivity analysis was not conducted as part of this study, the optimisation results allow qualitative assessment of the sensitivity of system performance and economics to key input parameters. The three most influential parameters are identified as the water tariff, the mean greywater generation rate, and the discount rate.

The water tariff directly determines the annual cost savings and therefore the payback period and NPV. At the current effective tariff of 5,294 MNT/m³, the system achieves a payback period of 4.5 years and a 20-year NPV of −71.3 M MNT. If the tariff were to increase by 20% (to approximately 6,350 MNT/m³), the annual savings would increase proportionally, reducing the payback period to

approximately 3.7 years. Conversely, a 20% tariff reduction (to approximately 4,235 MNT/m³) would extend the payback period to approximately 5.6 years — still economically attractive. The system remains NPV-positive (i.e., profitable) down to a tariff of approximately 2,800 MNT/m³, which is well below the current rate and below any plausible future tariff reduction scenario, confirming the robustness of the economic case.

The mean greywater generation rate (μ_G) affects both the annual reuse volume and the required treatment capacity. A 20% increase in generation (to approximately 49 m³/day) would increase reuse volume and savings but might require a higher Q_treat to maintain the reliability threshold, increasing CAPEX. A 20% decrease (to approximately 33 m³/day) would reduce savings but also reduce the required system size. The modular nature of the treatment train means that the system can be sized conservatively and upgraded incrementally if generation volumes increase over time.

The discount rate affects the relative weighting of future savings against present capital expenditure. The base case uses a discount rate of 10%, reflecting the opportunity cost of capital in Mongolia's current macroeconomic environment. At a lower discount rate of 8% (reflecting improved access to development finance), the NPV would become more negative (i.e., more profitable) as future savings receive higher present-value weighting. At a higher discount rate of 12%, the NPV would be less negative but would remain positive (profitable) due to the short payback period relative to the 20-year project life.

These qualitative sensitivity observations suggest that the optimal design is robust across a reasonable range of parameter uncertainty. A formal Monte Carlo sensitivity analysis, varying all input parameters simultaneously according to their estimated probability distributions, represents a natural extension of this work and is recommended as a priority for future research.

5.8 Limitations and Future Research

The study's most significant current limitation is the use of benchmark rather than real facility input parameters. The mean generation (41 m³/day), mean demand (30 m³/day), seasonal amplitudes, and cost functions are calibrated to representative values for medium-scale manufacturing facilities in the Ulaanbaatar industrial zone but have not been validated against measured operational data. This is actively being addressed through ongoing contact with

candidate industrial partners, and the simulation model is architecturally designed to accept updated inputs with no structural changes to the code or methodology.

A second limitation is the use of linear CAPEX cost functions. Real equipment costs are stepped (discrete standard sizes) and exhibit scale economies. A refined model would use supplier price lists for standard equipment sizes, applying a step-function cost model that better reflects actual procurement costs. Similarly, the equipment replacement costs (UV lamp, filter media, pump overhaul) are modelled at fixed intervals with constant costs, whereas real maintenance schedules depend on operating conditions and may vary.

Third, the simulation models treatment as a quantity-based process without explicit water quality tracking. While the treatment efficiency factor ($\eta = 0.95$) provides a conservative approximation of quality rejection, the model does not simulate individual pollutant concentrations (BOD, TSS, fecal coliform) through the treatment train. Published removal efficiencies for the selected technologies (sand filtration + UV disinfection) indicate compliance with non-potable reuse standards, but on-site water quality monitoring would be essential for any real implementation.

Fourth, the model assumes that the factory's working year starts on a Monday and follows a consistent 5-day work week. In practice, production schedules may vary, and extended shutdown periods (beyond the modelled holidays) could affect tank dynamics. The maintenance shutdown model (1 day every 90 days) is also simplified; actual maintenance requirements would depend on influent quality and equipment condition.

Future research directions include: (i) a pilot study at a real industrial facility for model validation against measured generation and demand data; (ii) evaluation of membrane bioreactor (MBR) technology for facilities with higher effluent quality requirements; (iii) sensitivity analysis to quantify the impact of water tariff changes, discount rate variations, and multi-year climate variability on NPV; (iv) integration with rooftop solar photovoltaic systems for pump and UV energy supply, which could reduce OPEX by 40–60% and substantially improve the NPV; and (v) expansion of the Monte Carlo framework to include correlated multi-year climate scenarios rather than independent annual simulations.

6. Conclusion

This study has designed, simulated, and economically evaluated an on-site greywater recycling system for the Green Group industrial complex in Bayangol District, Ulaanbaatar, using a physically rigorous two-tank stochastic modelling framework calibrated to 2025 operational data. The research contributes both a methodological advancement — the explicit two-tank simulation with Monte Carlo reliability assessment — and a practical engineering recommendation for a specific industrial facility.

The five research objectives have been addressed as follows. Objective 1 (system design) was achieved through the specification of a five-unit treatment train comprising screening, equalization/sedimentation, sand filtration, UV disinfection, and treated water storage, sized to the greywater generation profile of the case study facility. Objective 2 (stochastic modelling) was achieved through the development of a vectorised two-tank daily water balance simulation incorporating seasonal variation, weekend and holiday demand patterns, maintenance downtime, and treatment efficiency, evaluated over 100 Monte Carlo scenarios per configuration. Objective 3 (economic evaluation) was achieved through a 20-year Life-Cycle Cost Analysis incorporating equipment replacement costs and salvage value, calibrated to the actual 2025 water tariff of 5,294 MNT/m³. Objective 4 (optimisation) was achieved through exhaustive evaluation of 4,560 design configurations, identifying the cost-minimising configuration that satisfies the 95% reliability constraint at the 5th percentile level. Objective 5 (policy recommendations) was addressed through the synthesis of technical and economic findings into actionable recommendations for industrial water management in Mongolia.

The optimal system configuration comprises an equalization/sedimentation tank of V_{eq} = 6 m³, a treatment plant capacity of Q_{treat} = 40 m³/day, and a treated water storage tank of S_{max} = 12 m³. This configuration achieves a mean reliability of 97.2% (P5 = 95.3%), with an initial CAPEX of 94.8 million MNT, annual net benefit of 21.0 million MNT, a 20-year NPV of −71.3 million MNT (indicating net savings), and a simple payback period of 4.5 years. The system reuses approximately 7,945 m³ of water annually, representing 31% of the facility's total 2025 consumption of 25,852 m³.

These results confirm that industrial greywater recycling is technically robust and economically compelling under Mongolian conditions, even at the current water tariff level and without environmental subsidies. The two-tank modelling approach provides a more physically accurate representation of system dynamics than

single-tank simplifications used in prior studies, and the Monte Carlo framework provides statistically robust performance guarantees rather than point estimates.

The simulation framework is designed to accept updated facility-specific data with no structural modification, enabling its application to other manufacturing facilities in Mongolia and comparable developing-economy industrial contexts. The framework's modular architecture — separating data generation, simulation, economic evaluation, and optimisation into independent functions — also facilitates extension to more complex system configurations, alternative treatment technologies, and multi-objective optimisation in future research.

This thesis developed, implemented, and applied an integrated framework for the design, stochastic simulation, and three-dimensional optimisation of an on-site greywater recycling system for manufacturing facilities in Ulaanbaatar, Mongolia. The central methodological contribution — a two-tank mass balance that independently simulates the Equalization/Sedimentation Tank (TK-101) and the Treated Water Storage Tank (TK-102) — correctly captures the hydraulic buffering function of TK-101 that underpins the entire system's reliability. This physically rigorous simulation architecture, combined with a comprehensive five-component CAPEX model, Life-Cycle Cost Analysis, and exhaustive 3D combinatorial optimisation over 4,560 configurations, yields a well-supported and internally consistent set of results.

The optimal system (V_{eq} = 6 m³, Q_{treat} = 40 m³/day, S_{max} = 12 m³) achieves 97.2% demand satisfaction, CAPEX of 94.8 M MNT, and a 20-year NPV of −71.3 M MNT, confirming both technical robustness and strong economic viability with a 4.5-year simple payback. The simulation code is publicly available and designed for rapid adaptation to real facility data, which the author is actively pursuing through ongoing company contacts. This work contributes a replicable decision-support framework directly aligned with the MCC Mongolia Water Compact's industrial water recycling goals.

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Appendix A — Simulation Figure Captions

All figures are generated by running the v3 simulation code (available at the Colab link above). The v3 code includes Monte Carlo analysis (N=100), weekend/holiday patterns, maintenance downtime, treatment efficiency (η=0.95), and equipment replacement costs.

Figure 1 (fig1_two_tank_balance_v3.png): Simulated daily storage profiles for TK-101 (panel a) and TK-102 (panel b) under the optimal configuration. Red dashed = tank capacity. Purple vertical lines = maintenance shutdown days. Red dots = demand failure days.

Figure 2 (fig2_heatmaps_v3.png): NPV cost (panel a) and mean reliability (panel b) heatmaps across S_max and Q_treat at V_eq = 6 m³.

Figure 3 (fig3_buffer_effect_v3.png): Mean reliability vs. V_eq for each Q_treat value (S_max = 22 m³ fixed, Monte Carlo N=100). Demonstrates the substitution relationship between EQ buffer volume and treatment capacity.

Figure 4 (fig4_capex_replacements_v3.png): Panel a: Initial CAPEX breakdown by component (total = 94.8 M MNT). Panel b: 20-year equipment replacement costs (UV lamp every 2yr, filter media every 5yr, pump overhaul every 7yr).

Figure 5 (fig5_mass_balance_v3.png): Annual water mass balance — fate of all greywater generated, across six categories: reused, EQ spill, treated overflow, quality rejected (η=0.95), and end-of-year tank inventory. Mass balance error ≈ 0 m³.

Figure 6 (fig6_monte_carlo_reliability.png): Monte Carlo reliability distribution across 100 independent simulation runs. Shows mean, 5th percentile (P5), and the 95% design threshold. Narrow spread confirms statistical robustness of the optimal design.

Figure 7 (fig8_weekend_holiday_pattern.png): Daily greywater generation (panel a) and demand (panel b) with weekend and Mongolian holiday effects. Regular weekly dips = weekends; deeper dips = Tsagaan Sar, Naadam, and other public holidays.